Alexei Trofimov (Boston Univ.)
Gerco Onderwater (Univ. of Illinois at Urbana-Champaign)
Cenap Özben (Brookhaven Nat'l Lab.)

Muon (g-2) Measurement: The Multi-Parameter Fit Method

Analysis Strategy

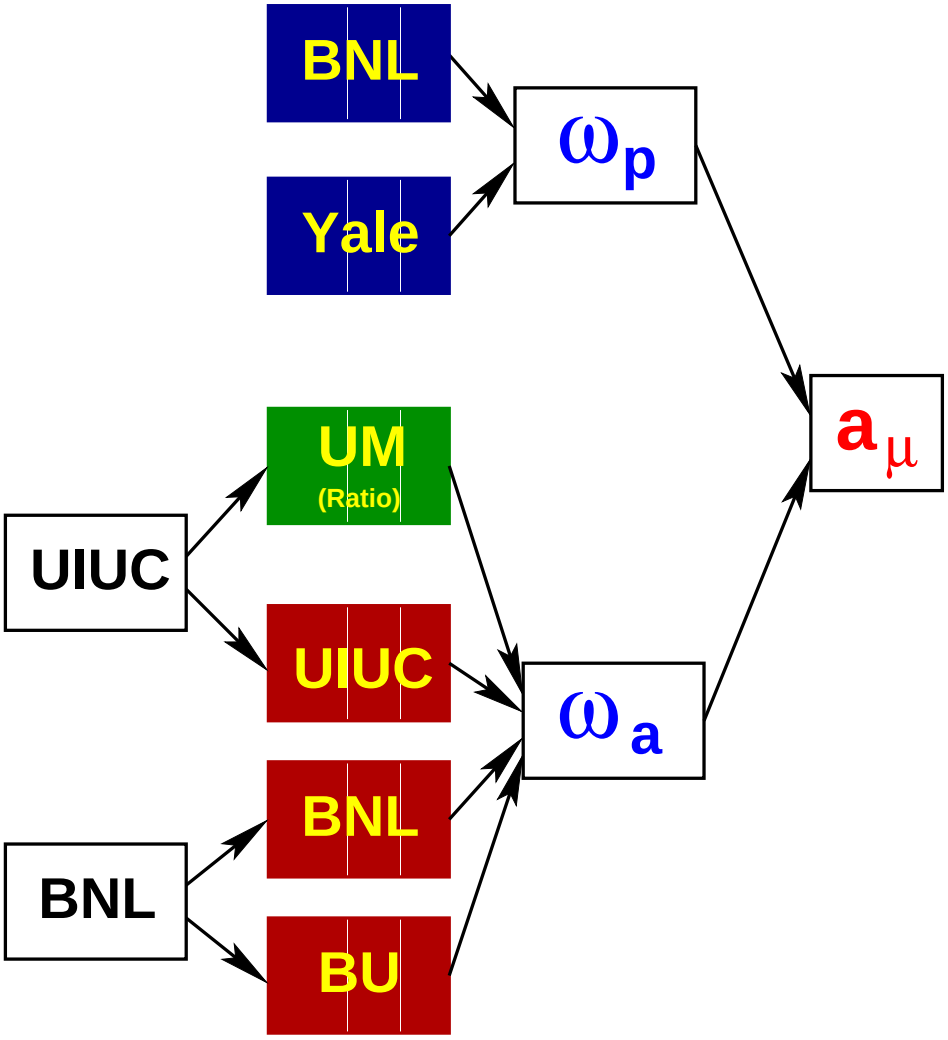
"Ideal" Function + Modifications:

- Pile-Up
- Muon Losses
- Betatron Oscillation
- Detector Gain Changes

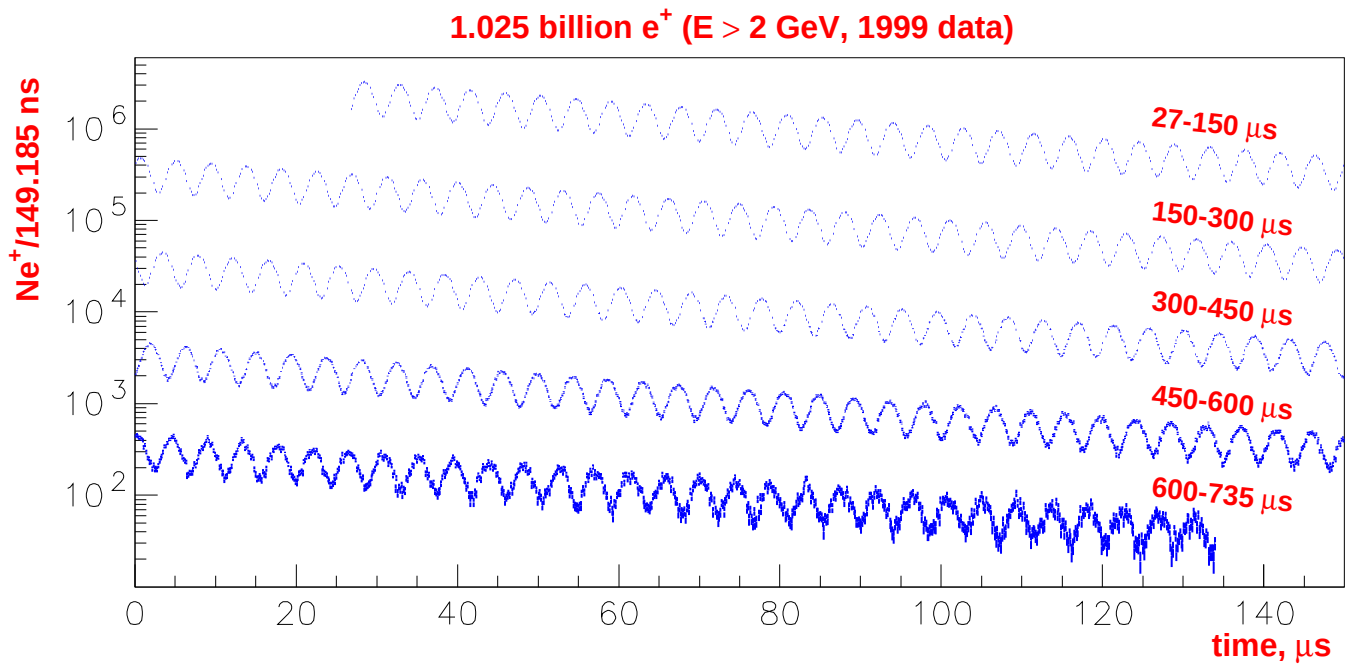
Fit Results

APS Meeting. April 28, 2001
Washington, DC

Analysis Strategy



- Two data production algorithms
- Analyses done with unknown offsets
- Offsets removed once the analyses are complete



Muon Decay + Spin Precession:

$$N(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

Additional Effects:

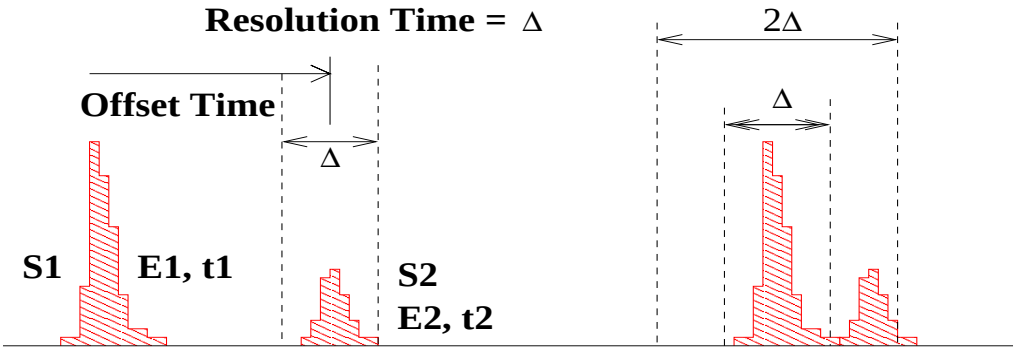
- Pile-Up
- Coherent Betatron Oscillation
- Muon Loss
- Detector Gain Changes

Pile-Up

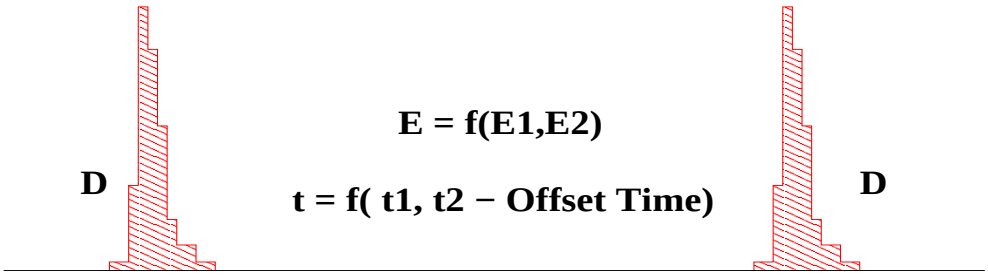
- Occurs when two pulses fall within the detector resolution time ($\sim$ 3 ns)
- Results in a distortion of the reconstructed pulse time, energy information
- Pile-up pulses carry "wrong" phase, asymmetry
- Number proportional to the rate squared and to the resolution time ($\sim$ 1% total)
- It is possible to approximate pile-up statistically using well-resolved pairs of pulses

BNL Method UIUC Method

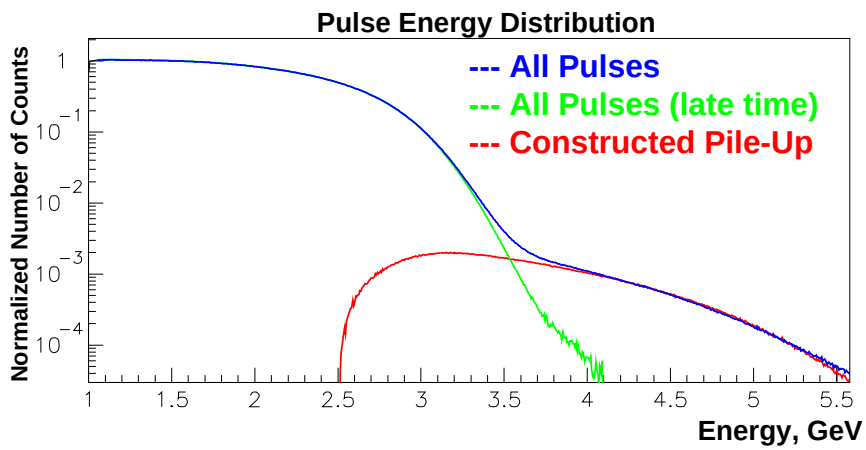
Pile−up Candidates



Constructed Doubles

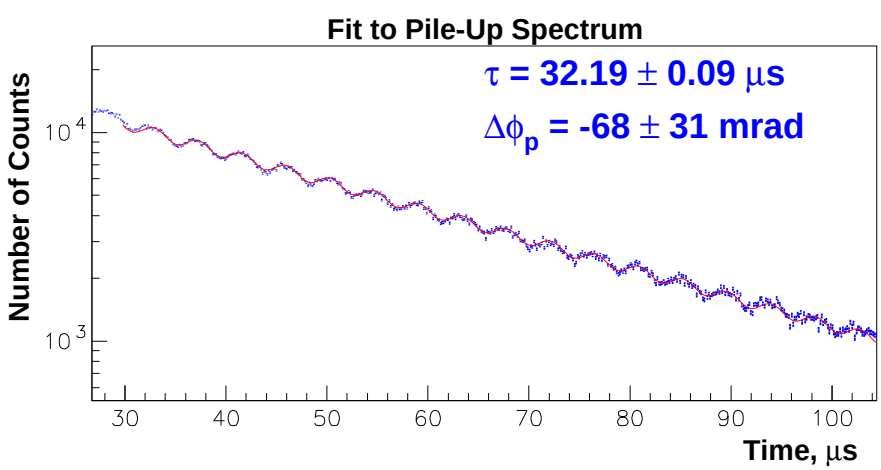


Correcting for Pile-Up



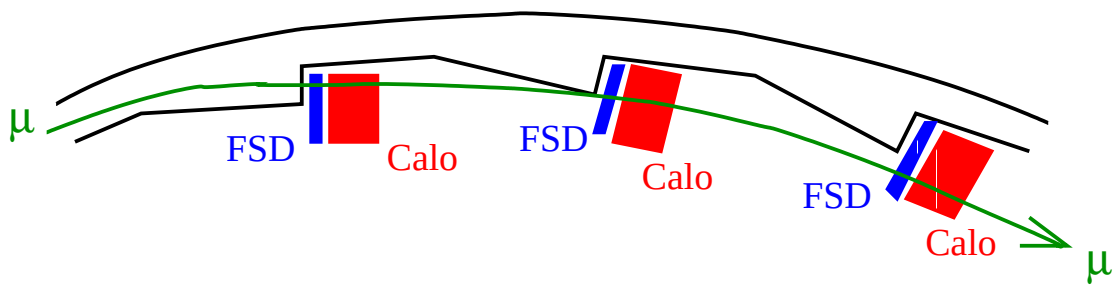
- BNL, UIUC: pile-up subtracted from the data using different construction methods
- BU: pile-up is fitted to, some parameters fixed after studies

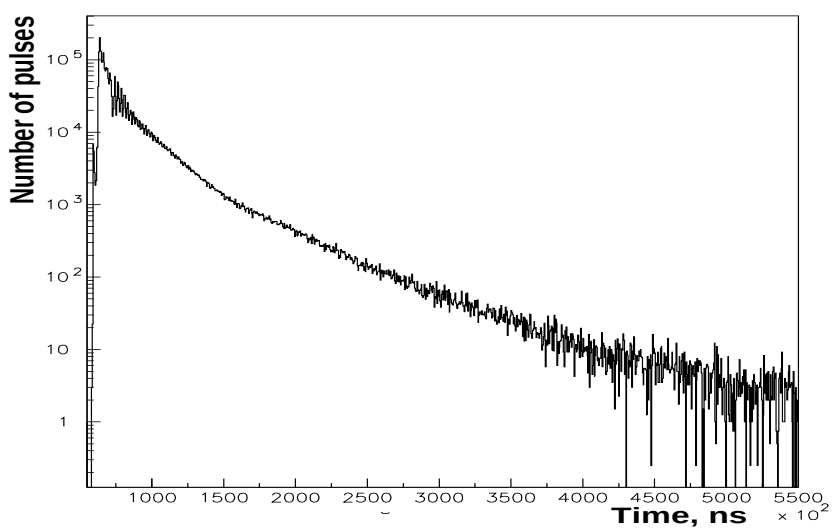
$$p(t) = N_0 e^{-2t/\lambda} \cdot (n_p + A_p \cdot \cos(\omega_a t + \phi + \Delta\phi_p)) \cdot \cdot (1 + a_p e^{-\frac{1}{2}(\frac{t}{\tau_p})^2})$$



Muon Losses

- Cause decay rate to decrease with time
- Can be seen in Front Scintillator Detectors
  (3-fold coincidences)





$$l(t) = 1 + a_{\mu L} \cdot e^{-(t/\tau_{\mu L})}$$

- BNL, BU: Muon losses are fitted
- UIUC: Fixed correction for muon losses
  (from studies)

Coherent Betatron Oscillation

- Inflector has a smaller aperture than the Storage Ring
- Non-ideal beam injection and kick

Beam centroid is moving,
beam width is changing

- Detector radial acceptances vary $\Rightarrow$ observed time distribution is affected
- Seen in Fourier Transform of the data, by Scintillator Fiber Detectors

$$b(t) = 1 + A_B \cdot e^{-(t/\tau_B)^2} \cdot \cos(\omega_B t + \phi_B)$$

Focusing Field Index: $n = -\frac{\rho}{\beta B}\frac{\partial E_r}{\partial r} \simeq 0.135$

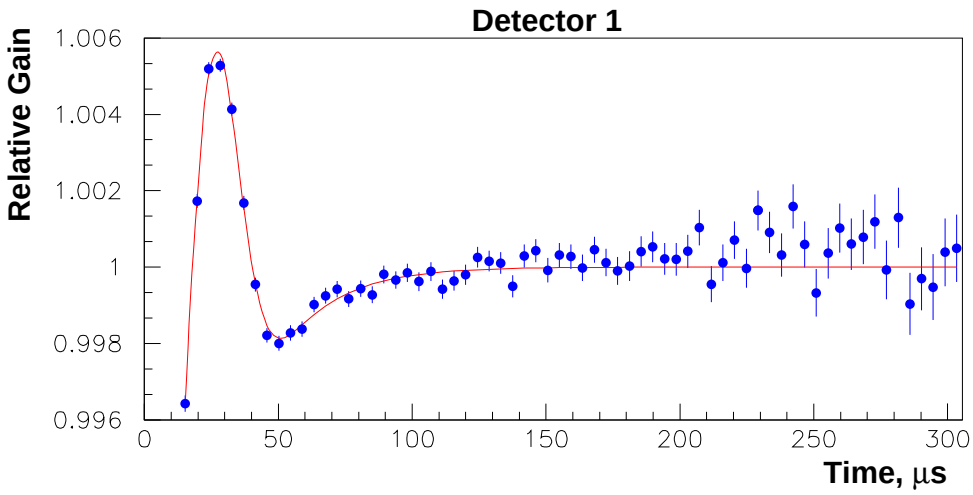
$$\omega_B = f_c(1 - \sqrt{1-n}) \simeq 470 \text{ kHz} \simeq 2\omega_a$$

- BNL: ω_B is fixed in the fit after studies

  UIUC: ω_B and τ_B are fixed

  BU: all free parameters

Detector Gain Changes



- Caused by instabilities in calorimeter PM tubes
- Can be determined from the average energy studies and the laser calibration system
- The effect is reduced significantly for sum of data from all detectors (coincidence)
- BNL: No correction used

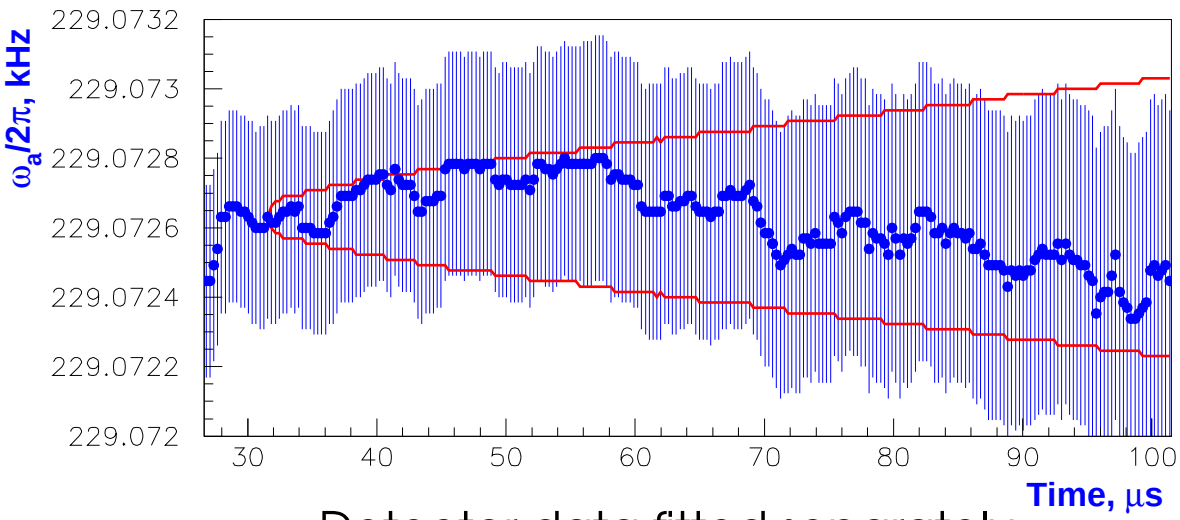
  BU: Use time-dependent energy threshold in data selection

  UIUC: Gain changes are parametrized in the fit function

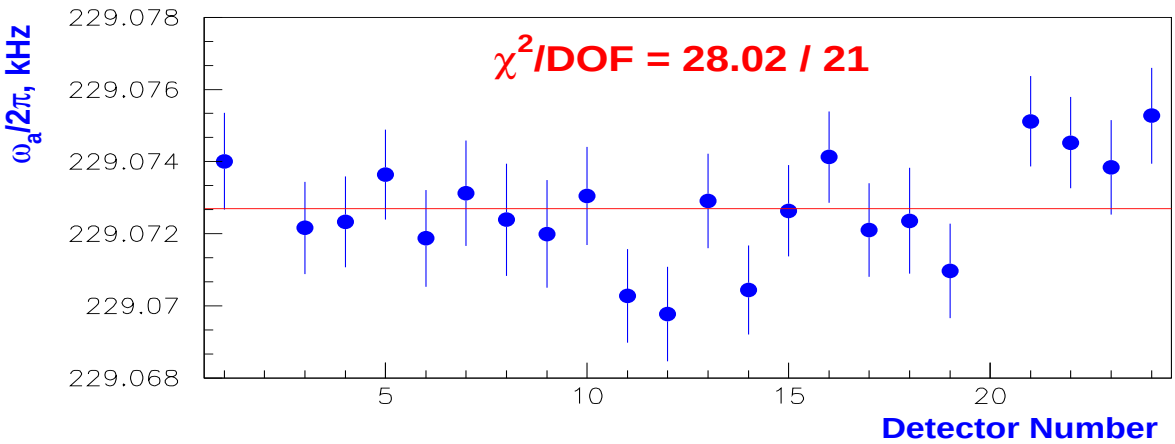
Fit Function and Results

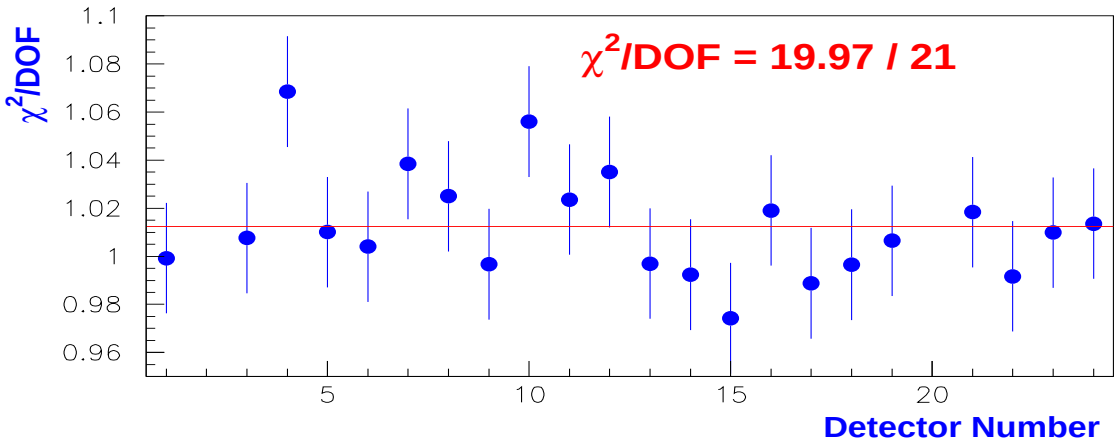
$$f(t) = \left(N_0 e^{-\lambda t} \cdot [1 + A\cos(\omega_a t + \phi)] + p(t)\right) \cdot b(t) \cdot l(t)$$

Data from 22 detectors combined ($\sigma_{ij} = \sqrt{\sigma_i^2 - \sigma_j^2}$)

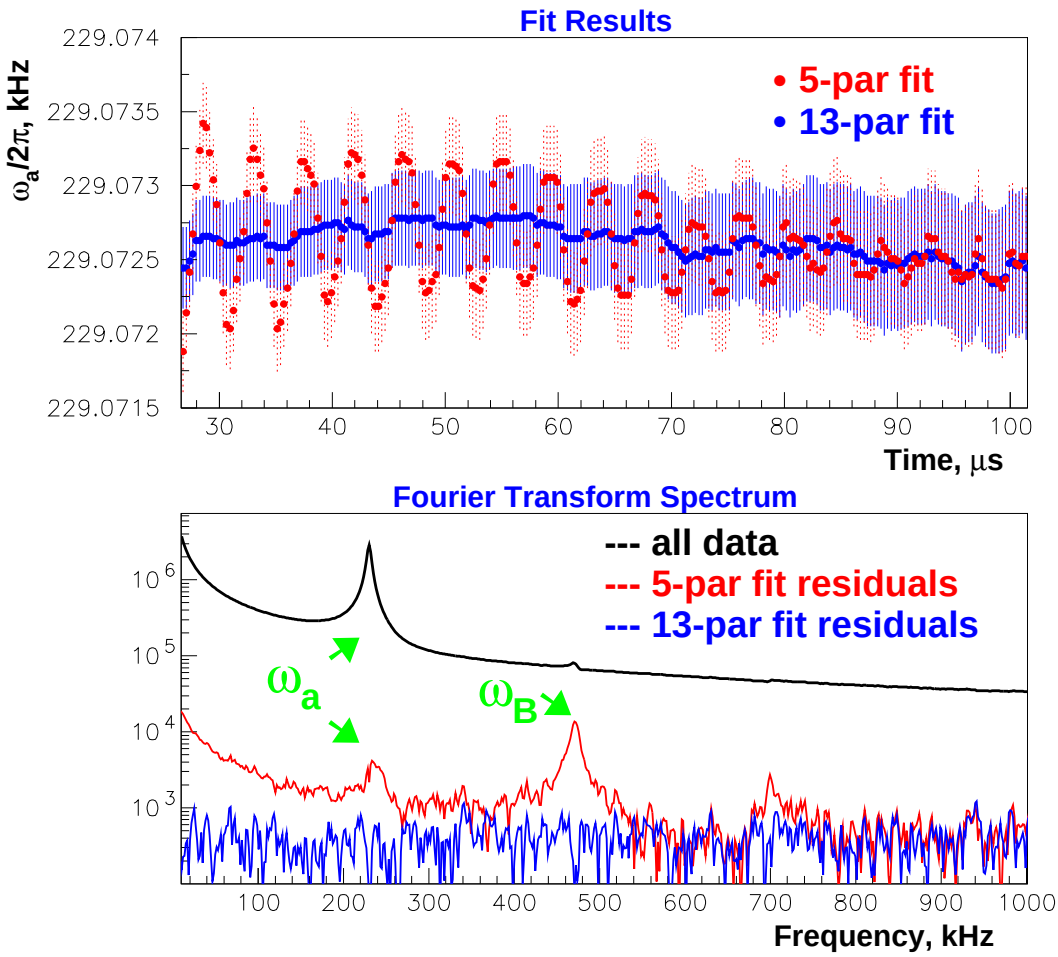


Detector data fitted separately:





5-par vs 13-par Fit



- Additional parameters are very weakly correlated to ω_a

Fit Results

Institute	Fit Start	Fit Par.	χ^2/DOF	R, ppm
BU	32 μs	13	1.012 $\pm$ 0.023	119.55 $\pm$ 1.24
BNL	32 μs	10	1.005 $\pm$ 0.023	119.38 $\pm$ 1.24
UIUC	25-56 μs	9	1.016 $\pm$ 0.005	119.60 $\pm$ 1.23
UM	34 μs	3	0.986 $\pm$ 0.025	119.67 $\pm$ 1.28

$\omega_a = 2\pi f_0 \cdot (1 - R \cdot 10^{-6})$, with f_0 = 229.1 kHz

- BU: CBO, Muon Losses, Pile-up (ϕ_p from study)
- BNL: CBO, Muon Losses, Pile-up corrected
- UIUC: CBO, Muon Losses fixed, Pile-up corrected
- UM: Ratio method

98-99% overlap in the analyzed data sets
Results agree within standard statistical tests

Fit Dependent Systematic Errors (in ppm)

Source	BU	BNL	UIUC	UM
Pile-up, Muon Losses, Gain	0.14	0.17	0.09	0.16
Background	0.12	0.08	0.10	0.08
Betatron Oscillations	0.09	0.11	0.06	0.08
Fitting Method	0.08	0.08	0.05	0.08
Randomization	0.04	0.04	0.03	0.05
Total	0.22	0.24	0.16	0.19

Common Systematics: 0.18 ppm

Total Systematic Error on ω_a: 0.3 ppm

Summary

- The data from 1999 run, whether corrected for pile-up or not, can be well described by a multi-parameter function.

- χ^2/DOF = 3818 / 3799 (BNL analysis)

- The results of different analyses agreed to 0.3 ppm (within expected statistical variation)

- Final result is a weighted average, accounting for strong correlation in the data sets.